ALGEBRAIC MANIPULATION CHALLENGE – SOLUTIONS

PROBLEM #1

FACTS: $\chi = 4.00\ \Psi°$ $\chi = \dfrac{6.00\ r}{\theta\ \alpha^2}$ $\Psi° = \dfrac{27.0\ \rho}{\aleph}$

DATA: $r = 0.822\ \text{cm}^3$ $\rho = 1.06\ \text{cm}$ $\aleph = 3.011\ \text{g}$ $\theta = 42.3\ \text{g}$

Solve for α : (What are the final units?)

$$4.00\ \Psi° = \frac{6.00\ r}{\theta\ \alpha^2}$$

$$4.00\ \frac{27.0\ \rho}{\aleph} = \frac{6.00\ r}{\theta\ \alpha^2}$$

$$\alpha^2 = \frac{6.00\ r}{\theta} \cdot \frac{\aleph}{4.00 \cdot 27.0\ \rho}$$

$$\alpha^2 = \frac{6.00\ r\ \aleph}{\theta \cdot 4.00 \cdot 27.0\ \rho}$$

$$\sqrt{\alpha^2} = \sqrt{\frac{6.00\ r\ \aleph}{\theta \cdot 4.00 \cdot 27.0\ \rho}}$$

$$\alpha = \sqrt{\frac{6.00\ r\ \aleph}{\theta \cdot 4.00 \cdot 27.0\ \rho}}$$

Plug in data:

$$\alpha = \sqrt{\frac{6.00 \cdot \mathbf{0.822\ cm^3} \cdot \mathbf{3.011\ g}}{\mathbf{42.3\ g} \cdot 4.00 \cdot 27.0 \cdot \mathbf{1.06\ cm}}}$$

Cancel units:

$$\alpha = \sqrt{\frac{6.00 \cdot \mathbf{0.822\ cm^{\cancel{3}\,2}} \cdot \mathbf{3.011\ \cancel{g}}}{\mathbf{42.3\ \cancel{g}} \cdot 4.00 \cdot 27.0 \cdot \mathbf{1.06\ \cancel{cm}}}}$$

resulting units: $\sqrt{\text{cm}^2} = \text{cm}$

Numerical answer: $\boxed{\mathbf{5.54 \times 10^{-2}\ cm}}$

PROBLEM #2

$$\varphi\beta\pi = 3.0\left[\frac{(8.6\rho)}{\delta}\right]$$

$$\frac{1}{\sqrt{f\Delta}}\,\beta\pi = 3.0\left[\frac{(8.6\rho)}{\delta}\right]$$

$$\frac{1}{\frac{\sqrt{2.6\,\Delta}}{\omega}}\,\beta\pi = 3.0\left[\frac{(8.6\rho)}{\delta}\right]$$

Solve for Δ:

$$\frac{\beta\pi}{3.0\left[\frac{(8.6\rho)}{\delta}\right]} = \frac{\sqrt{2.6\,\Delta}}{\omega}$$

$$\left(\frac{\beta\pi}{3.0\left[\frac{(8.6\rho)}{\delta}\right]}\right)^2 = \left(\frac{\sqrt{2.6\,\Delta}}{\omega}\right)^2$$

$$\left(\frac{\beta\pi}{3.0\left[\frac{(8.6\rho)}{\delta}\right]}\right)^2 = \frac{2.6\,\Delta}{\omega}$$

$$\frac{\omega}{2.6}\left(\frac{\beta\cdot\pi}{3.0\left[\frac{(8.6\rho)}{\delta}\right]}\right)^2 = \Delta$$

FACTS:

$$\varphi\beta\pi = 3.0\tau \qquad \tau = \frac{8.6\rho}{\delta} \qquad f = \frac{2.6}{\omega}$$

$$\varphi = \frac{1}{\sqrt{f\Delta}} \qquad \pi = \pi$$

DATA:

$$\omega = 2.1\ kg \qquad \beta = 15.01\ \frac{kg}{s}$$

$$\delta = 0.60\ m \qquad \rho = 20.0\ kg$$

Plug in data: (note: $\pi = \pi$)

$$\frac{2.1\ kg}{2.6}\left(\frac{15.0\ \frac{kg}{s}\cdot 3.14}{3.0\left[\frac{8.6\cdot 20.0\ kg}{0.60\ m}\right]}\right)^2 = \Delta$$

Resulting units: $kg\cdot\frac{kg^2}{s^2}\cdot\frac{m^2}{kg^2} = \frac{kg\,m^2}{s^2}$

Numerical answer: $0.0024\ \frac{kg\,m^2}{s^2}$